9. G_9-ESTIMATOR OF THE SOLUTION OF THE DISCRETE KOLMOGOROV-WIENER FILTER

The discrete analog of the Kolmogorov-Wiener filter has the form

$$R_m\vec{\varphi}=\vec{b}, \tag{9.1}$$

where

$$R_m=\left\{m^{-1}R\left(sm^{-1},km^{-1}\right)\right\}_{k,s=1}^{m};\ \ \vec{b}^T\left(t\right)=\left\{Q\left(t,sm^{-1}\right),\ \ s=1,\dots,m\right\},$$

$$\vec{\varphi}^T\left(t\right)=\left\{\varphi\left(t,km^{-1}\right),\ \ k=1,...,m\right\},$$

$$R\left(x,y\right)=\mathbf{E}\left[\alpha\left(x\right)-\mathbf{E}\,\alpha\left(x\right)\right]\left[\alpha\left(y\right)-\mathbf{E}\,\alpha\left(y\right)\right],$$

$$Q\left(x,y\right)=\mathbf{E}\left[\alpha\left(x\right)-\mathbf{E}\,\alpha\left(x\right)\right]\left[\beta\left(y\right)-\mathbf{E}\,\beta\left(y\right)\right],$$

and $\alpha\left(x\right),\ \beta\left(y\right)$ are random processes. If $R_m>0$, then the estimator $\hat{\vec{\varphi}}=\left(\hat{R}_m\right)^{-1}\hat{\vec{b}}$ converges in probability to $\vec{\varphi}$ when $n_1,\ n_2\to\infty$, where

$$\hat{R}=\left\{m^{-1}\hat{R}\left(sm^{-1},km^{-1}\right)\right\}_{k,s=1}^{m},\ \ \vec{\varphi}^T\left(t\right)=\left\{\varphi\left(t,km^{-1}\right),\ \ k=1,...,m\right\};$$

$$\hat{\vec{b}}^T\left(t\right)=\left\{\hat{Q}\left(t,sm^{-1}\right),\ \ s=1,\dots,m\right\},$$

$$\hat{R}\left(x,y\right)=\left(n_1-1\right)^{-1}\sum_{k=1}^{n_1}\left[\alpha_k\left(x\right)-\hat{\alpha}\left(x\right)\right]\left[\alpha_k\left(y\right)-\hat{\alpha}\left(y\right)\right],$$

$$\hat{Q}\left(x,y\right)=\left(n_2-1\right)^{-1}\sum_{k=1}^{n_2}\left[\alpha_k\left(x\right)-\hat{\alpha}\left(x\right)\right]\left[\beta_k\left(y\right)-\hat{\beta}\left(y\right)\right],$$

and $\alpha_k\left(x\right),\ \beta_k\left(y\right)$ are independent observations of $\alpha\left(x\right),\ \beta\left(y\right)$.

As mentioned in previous sections of this chapter, the large order of system (9.1) requires a large number of observations of stochastic processes $\alpha\left(x\right),\ \beta\left(y\right)$.

Therefore, it is of interest to obtain more accurate estimators. Applying the G-analysis technique, which is described in [Gir44, Gir54, Gir69, Gir84], we can obtain an estimator of $\vec{\varphi}$, such that it would approach in probability $\vec{\varphi}$, provided that $\lim_{n\to\infty}mn^{-1}=c<1$. We assume for simplification of formulas that vector $\vec{b}$ is known. This estimator will be referred to as the G_9-estimator. It is

$$\vec{G}_9=\left(\hat{R}_m\right)^{-1}\vec{b}\left(1-\frac{m_n}{n}\right). \tag{9.2}$$

Denote

$$\vec{\alpha}_k^{\mathrm{T}}=\left(\alpha_k\left(\frac{s}{m}\right),\ s=1,\dots,m\right),\qquad R^{-1/2}(\vec{\alpha}_k-\mathbf{E}\,\vec{\alpha}_k)=\vec{\xi}_k=(\xi_{sk}\ \ s=1,\dots,m)^T.$$

THEOREM 9.1. ([Gir44], [Gir54], [Gir69], [Gir84]) *If random variables* ξ_{sk} *are independent for every* n, $\mathbf{E}\left|\xi_{ki}\right|^{4+\delta} \leq c,\ \delta > 0,\ \lim_{n_1\to\infty} mn_1^{-1} < 1;\ \lambda_i\left(R_m\right) \leq c < \infty$, *the vector* $\vec{b}$ *is known,*

$$\sup_m \left[\vec{b}^T\vec{b} + \vec{c}^T\vec{c}\right] < \infty,$$

where $\vec{c} \in R^m$, *and* $\lambda_i\left(R_m\right)$ *are the eigenvalues of the matrix* R_m, *then*

$$p\lim_{n_1\to\infty} \left[\vec{c}^T\vec{G}_9 - \vec{c}^T\vec{\varphi}\right] = 0.$$

Random Matrices. Kiev University Publishing, Ukraine, 1975, 448pp. (in Russian).

Theory of Random Determinants. Kiev University Publishing, Ukraine, 1975, 368pp. (in Russian).

Limit Theorems for Functions of Random Variables. "Higher School" Publishing, Kiev, Ukraine, 1983, 207pp. (in Russian).

Multidimensional Statistical Analysis. "Higher School" Publishing, Kiev, Ukraine, 1983, 320pp. (in Russian).

Spectral Theory of Random Matrices. "Science" Publishing, Moscow, Russia, 1988, 376pp. (in Russian).

Theory of Random Determinants. Kluwer Academic Publishers, The Netherlands, 1990, 677pp.

Theory of Systems of Empirical Equations. "Lybid" Publishing, Kiev, Ukraine, 1990, 264pp. (in Russian).

Statistical Analysis of Observations of Increasing Dimensions. Kluwer Academic Publishers, The Netherlands, 1995, 286pp.

Theory of Linear Algebraic Equations with Random Coefficients. Allerton Press, Inc, New York, U.S.A. 1996, 320pp.

An Introduction to Statistical Analysis of Random Arrays. VSP, Utrecht, The Netherlands. 1998, 673pp.

Theory of Stochastic Canonical equations. Volume I and II. Kluwer Academic Publishers, (xxiv + 497, xviii + 463).(2001).