3. G_3-ESTIMATOR OF INVERSE COVARIANCE MATRIX

The G_3-estimator of a matrix $R_{m_n}^{-1}$ is equal to

$$G_3 = \hat{R}_{m_n}^{-1} \left[1 - m_n n^{-1} \right].$$

THEOREM 3.1. ([Gir44], [Gir54]) *If G-condition* $\limsup_{n\to\infty} m_n n^{-1} < 1$ *is fulfilled, components* $\xi_{ik},\ i = 1, \ldots, m_n$ *of the vectors*

$$\vec{\xi}_k = \{\xi_{ik},\ i = 1, \ldots, m_n\}^T = R_{m_n}^{-1/2} \left[\vec{x}_k - \vec{a}_k\right],\ k = 1, \ldots, n$$

are independent and for some $\delta > 0$

$$\sup_n \max_{\substack{i=1,\ldots,m_n;\\ k=1,\ldots,n}} \mathbf{E} \left|\xi_{ik}\right|^{4+\delta} < \infty,$$

$$\vec{b}^T \vec{b} < c_1, \quad \vec{a}^T \vec{a} < c_2, \quad 0 < c_3 < \lambda_{\min}\left(R_{m_n}\right) \leq \cdots \leq \lambda_{\max}\left(R_{m_n}\right) \leq c_4,$$

then

$$\underset{n\to\infty}{p\lim} \left[\vec{a}^T G_3 \vec{b} - \vec{a}^T R_{m_n}^{-1} \vec{b}\right] = 0.$$

THEOREM 3.2. ([Gir44], [Gir54]) *If G-condition* $\limsup_{n\to\infty} m_n n^{-1} < 1$ *holds, components* ξ_{ik}, $i = 1, \ldots, m_n$ *of the vectors*

$$\vec{\xi}_k = \{\xi_{ik},\ i = 1, \ldots, m_n\}^T = R_{m_n}^{-1/2} \left[\vec{x}_k - \vec{a}_k\right],\ k = 1, \ldots, n$$

are independent, have the standard Normal distribution and

$$\vec{b}^T \vec{b} < c_1, \quad , \vec{a}^T \vec{a} < c_2, \quad \lambda_{\min}\left[R_{m_n}\right] > c_3 > 0,$$

then

$$\lim_{n\to\infty} \mathbf{P}\left\{\left[\vec{a}^T G_3 \vec{b} - \vec{a}^T R_{m_n}^{-1} \vec{b}\right] c_n < x\right\} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} \exp\left\{-y^2/2\right\} \mathrm{d}y,$$

where c_n *is a certain sequence of constant.*

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