17. G_{17}-ESTIMATE OF T^2 -STATISTICS

The multi-dimensional analogue of Student's t^2 statistics is

$$T^2 := \left(\vec{a} - \hat{\vec{a}}\right)^T \hat{R}_{m_n}^{-1} \left(\vec{a} - \hat{\vec{a}}\right)$$

where

$$\hat{R}_{m_n} = (\hat{r}_{ij})_{i,j=1}^{m_n} = n^{-1} \sum_{k=1}^{n} (\vec{x}_k - \hat{\vec{a}})(\vec{x}_k - \hat{\vec{a}})^T, \;\; \hat{\vec{a}} = n^{-1} \sum_{k=1}^{n} \vec{x}_k,$$

$\vec{x}_k, k = 1, ..., n$ are independent observations of a vector $\vec{\xi}$,

$$\mathbf{E}\,\vec{\xi} = \vec{a}, \quad \mathbf{E}\, \left(\vec{\xi} - \vec{a}\right) \left(\vec{\xi} - \vec{a}\right)^T = R_m.$$

From [Gir69, p.146] we obtain the limit of random variable T^2 when random vectors $\vec{x}_k - \vec{a}$; $k = 1, \ldots, n$ are independent and G-condition is fulfilled.

THEOREM 17.1. ([Gir69, p.146]*If G-condition* $\limsup_{n\to\infty} mn^{-1} < 1$ *is fulfilled, components* $\xi_{ik}, \; i = 1, \ldots, m$ *of the vectors*

$$\vec{\xi}_k = \{\xi_{ik}, \; i = 1, \ldots, m\}^T = R_{m_n}^{-1/2} \left[\vec{x}_k - \vec{a}_k\right], \; k = 1, \ldots, n$$

are independent and for some $\delta > 0$

$$\sup_n \max_{\substack{i=1,\ldots,m; \\ k=1,\ldots,n}} \mathbf{E}\, |\xi_{ik}|^{4+\delta} < \infty,$$

$$\vec{a}^T \vec{a} < c_2, \quad \lambda_{\min} \left[R_{m_n}\right] > c_3 > 0,$$

then

$$p\lim_{n\to\infty} \left| \left(1 - mn^{-1}\right) \left(\vec{a} - \hat{\vec{a}}\right)^T \hat{R}_{m_n}^{-1} \left(\vec{a} - \hat{\vec{a}}\right) - mn^{-1} \right| = 0.$$

We call the expression $\left(1 - mn^{-1}\right) \left(\vec{a} - \hat{\vec{a}}\right)^T \hat{R}_{m_n}^{-1} \left(\vec{a} - \hat{\vec{a}}\right) - mn^{-1}$ G_{17}–estimate of T^2-statistics.

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