15. G_{15} -ESTIMATOR OF THE NONLINEAR DISCRIMINANT FUNCTION, OBTAINED BY OBSERVATION OF RANDOM VECTORS WITH DIFFERENT COVARIANCE MATRICES

In the case of classifying into two populations based on the Normal distribution, the nonlinear discriminant function is equal to

$$V(\vec{x}) = \frac{1}{2}\left\{-(\vec{x}-\vec{a}_1)^T R_1^{-1}(\vec{x}-\vec{a}_1) + (\vec{x}-\vec{a}_2)^T R_2^{-1}(\vec{x}-\vec{a}_2) - \ln\det R_1 R_2^{-1}\right\}.$$

Let $\vec{x}_i,\ \vec{y}_j;\ i=1,\ldots,n_1;\ j=1,\ldots,n_2$ be independent observations of m-dimensional random vectors $\vec{\xi}$ and $\vec{\eta}$ respectively; these vectors $\vec{\xi}$ and $\vec{\eta}$ are independent and distributed as $N\{\vec{a}_1,\ R\},\ N\{\vec{a}_2,\ R\}$. As the empirical mean vectors and the covariance matrices R_i, we take:

$$\hat{\vec{a}}_1 = n_1^{-1}\sum_{i=1}^{n_1}\vec{x}_i,\ \hat{\vec{a}}_2 = n_2^{-1}\sum_{i=1}^{n_2}\vec{y}_i,$$

$$\hat{R}_1 = \frac{1}{n_1-1}\sum_{i=1}^{n_1}\left(\vec{x}_i-\hat{\vec{a}}_1\right)\left(\vec{x}_i-\hat{\vec{a}}_1\right)^T,\ \hat{R}_2 = \frac{1}{n_2-1}\sum_{i=1}^{n_2}\left(\vec{y}_i-\hat{\vec{a}}_2\right)\left(\vec{y}_i-\hat{\vec{a}}_2\right)^T.$$

We shall refer to the expression

$$\frac{1}{2}\left[-\left(\vec{x}-\hat{\vec{a}}_1\right)^T\hat{R}_1^{-1}\left(\vec{x}-\hat{\vec{a}}_1\right)\frac{n_1-1-m}{n_1-1}+\frac{m}{n_1}\right.$$
$$\left.+\left(\vec{x}-\hat{\vec{a}}_2\right)^T\hat{R}_2^{-1}\left(\vec{x}-\hat{\vec{a}}_2\right)\frac{n_2-1-m}{n_2-1}+\frac{m}{n_2}\right]-\frac{1}{2}\ln\frac{\det\hat{R}_1\hat{R}_2^{-1}}{\left(1-mn_1^{-1}\right)\left(1-mn_2^{-1}\right)}$$

as the $G_{15}(\vec{x})$ -estimator of the nonlinear discriminant function.

THEOREM 15.1. [Gir54, p.615] *Let random variables $\mu_i,\ \nu_i,\ i=1,\ldots,m$ be independent for every n, $\mathbf{E}\,\mu_i = \mathbf{E}\,\nu_i = 0,\quad \mathbf{E}\,\mu_i^2 = \mathbf{E}\,\nu_i^2 = 1,\quad i=1,\ldots,m,$ for a certain $\beta>0$*

$$\sup_n \max_{i=1,\ldots,m}\mathbf{E}\left[|\mu_i|^{4+\beta}+|\nu_i|^{4+\beta}\right]<\infty,$$

$$\inf_n \min_{i=1,\ldots,m}\lambda_i(R)>0,\quad \sup_n\max_{i=1,\ldots,m}\lambda_i(R)<\infty,$$

and the G-condition be satisfied:

$$\lim_{m\to\infty} mn_1^{-1}<1,\ \lim_{m\to\infty} mn_2^{-1}<1,$$

$$\sup_m(\vec{a}_1-\vec{a}_2)^T(\vec{a}_1-\vec{a}_2)<\infty.$$

Then,

$$\mathop{p\lim}_{n\to\infty} \{G_{15}(\nu) - V(\nu)\} = 0,$$

where ν is an observation of vector $\vec{\xi}$ or $\vec{\eta}$, which does not depend on $\vec{x}_i$, $\vec{y}_i$.

Random Matrices. Kiev University Publishing, Ukraine, 1975, 448pp. (in Russian).

Theory of Random Determinants. Kiev University Publishing, Ukraine, 1975, 368pp. (in Russian).

Limit Theorems for Functions of Random Variables. "Higher School" Publishing, Kiev, Ukraine, 1983, 207pp. (in Russian).

Multidimensional Statistical Analysis. "Higher School" Publishing, Kiev, Ukraine, 1983, 320pp. (in Russian).

Spectral Theory of Random Matrices. "Science" Publishing, Moscow, Russia, 1988, 376pp. (in Russian).

Theory of Random Determinants. Kluwer Academic Publishers, The Netherlands, 1990, 677pp.

Theory of Systems of Empirical Equations. "Lybid" Publishing, Kiev, Ukraine, 1990, 264pp. (in Russian).

Statistical Analysis of Observations of Increasing Dimensions. Kluwer Academic Publishers, The Netherlands, 1995, 286pp.

Theory of Linear Algebraic Equations with Random Coefficients. Allerton Press, Inc, New York, U.S.A. 1996, 320pp.

An Introduction to Statistical Analysis of Random Arrays. VSP, Utrecht, The Netherlands. 1998, 673pp.

Theory of Stochastic Canonical equations. Volume I and II. Kluwer Academic Publishers, (xxiv + 497, xviii + 463).(2001).