1. G_1-ESTIMATOR OF GENERALIZED VARIANCE

Let the independent observations $\vec{x}_1, ..., \vec{x}_n$ of the m_n-dimensional random vector $\vec{\xi}$, $n > m_n$ be given,

$$\hat{R} := (n-1)^{-1} \sum_{k=1}^{n} (\vec{x}_k - \hat{\vec{x}})(\vec{x}_k - \hat{\vec{x}})^{\mathrm{T}}, \quad \hat{\vec{x}} = n^{-1} \sum_{k=1}^{n} \vec{x}_k.$$

The expression $\det R$ is called a generalized variance. If the vectors $\vec{x}_i, \ i = 1, \ldots, n$ are independent and distributed according to the multidimensional Normal law $N(\vec{a}, R)$, then

$$\det \hat{R} \approx [\det R](n-1)^{-m} \prod_{i=n-m}^{n-1} \chi_i^2,$$

where χ_i^2 are independent random variables with the χ^2-distribution and i degrees of freedom. In the general case, the distribution of $\det \hat{R}$ is intractable, and therefore finding a consistent estimator for $\det R$ is a very complicated problem. It is proved (see [Gir39–41], [Gir43–45], [Gir53–55], [Gir 69], [Gir75]), that under certain conditions the G-estimators for $c_n^{-1} \ln \det R$ equal

$$G_1(\hat{R}) := c_n^{-1}\{\ln \det \hat{R} + \ln[(n-1)^m (A_{n-1}^m)^{-1} n(n-m_n)^{-1}\},$$

where $A_{n-1}^m = (n-1)...(n-m)$, c_n is a sequence of constants such that

$$\lim_{n\to\infty} c_n^{-2} \ln[n(n-m_n)^{-1}] = 0.$$

For every value $n > m_n$, let the m_n-dimensional random vectors $\vec{x}_1^{(n)}, ..., \vec{x}_n^{(n)}$ be independent and identically distributed with a mean vector $\vec{a}$ and nondegenerate covariance matrices R_{m_n}. For a certain $\delta > 0$

$$\sup_n \sup_{\substack{i=1,\ldots,n \\ j=1,\ldots,m_n}} \mathbf{E}\, |\tilde{x}_{ij}^{(n)}|^{4+\delta} < \infty,$$

where $\tilde{\vec{x}}_{ij}^{(n)}$ are the components of the vector $\tilde{\vec{x}}_i = R_{m_n}^{-1/2}(x_i^{(n)} - \vec{a})$, and

$$\lim_{n\to\infty} (n - m_n) = \infty, \quad \lim_{n\to\infty} n m_n^{-1} \geq 1;$$

and for each value of $n > m_n$, let the random variables $\tilde{x}_{ij}^{(n)}, \ i = 1, ..., n, \ j = 1, ..., m_n$ be independent. Then (see [Gir39–41], [Gir43–45], [Gir53–55], [Gir 69], [Gir75])

$$p \lim_{n\to\infty} [G_1(\hat{R}_{m_n}) - c_n^{-1} \ln \det R_{m_n}] = 0,$$

$$\lim_{n\to\infty} \mathbf{P}\{[c_n G_1(\hat{R}_{m_n}) - \ln \det R_{m_n}][-2\ln(1 - m_n n^{-1})]^{-1/2} < x\}$$
$$= (2\pi)^{-1/2} \int_{-\infty}^{x} \mathrm{e}^{-y^2/2}\, \mathrm{d}y.$$

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